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Moment conditions for fixed effects count data models with endogenous regressors

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Abstract

This note shows that moment conditions originally proposed by Wooldridge (1991) [Wooldridge, J.M., 1991. Multiplicative panel data models without the strict exogeneity assumption. Working Paper 574, MIT, Department of Economics] can be used for the consistent estimation of parameters in fixed effects count data models with endogenous regressors. © 2000 Elsevier Science S.A. All rights reserved.

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1. Introduction

This note considers the GMM estimation of panel count data models with multiplicative fixed effects with explanatory variables that are endogenous. Chamberlain (1992) proposed moment conditions for the consistent estimation by GMM of the parameters in a model with weakly exogenous regressors. These moment conditions have been further developed and extended to other models by Wooldridge (1997). They are valid when the explanatory variables are weakly exogenous, but not when these variables are endogenously determined, or when they are measured with error. In this note I will discuss moment conditions that are valid when the explanatory variables are endogenous. These are the same moment conditions as proposed by Wooldridge (1991). A problem of indeterminacy when one of the regressors is nonnegative or nonpositive can be circumvented by taking deviations from the overall mean.

Further, given a set of standard assumptions it will be shown that there are more moment conditions

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available than there are commonly used. These moment conditions are similar to those as described for the linear model by Ahn and Schmidt (1995).

2. Model and moment conditions

Let y_{it} be the dependent count variable taking values 0, 1, 2, ...; x_{it} an explanatory variable; i denotes individuals, $i = 1, \ldots, N$, and t denotes time, $t = 1, \ldots, T$. There are a large number of individuals N, but a fixed short time period T.

Consider the model

$$y_{it} = \exp(x_{it}\beta)u_{it}$$

$$u_{it} = \alpha_i v_{it},$$

and the following set of assumptions:

$$\alpha_i \text{ and } \alpha_i^2 \text{ are not correlated with } v_{it}$$

$$v_{it} \text{ is not correlated with } v_{is},\ s \neq t.$$

Further, the x process is correlated with α, and is either assumed to be weakly exogenous, $d = 1$, or endogenously determined, $d = 0$:

$$x_{it} \text{ is correlated with } v_{it-s},\ s \geq d$$

$$x_{it} \text{ is not correlated with } v_{it+j},\ j \geq 1 - d.$$

Then the following moment conditions hold:

$$E(x_{it-s}\,\Delta u_{it}) = 0,\ s \geq 2 - d \tag{1}$$

$$E(u_{it}\,\Delta u_{it-1}) = 0. \tag{2}$$

Let $\mu_{it} = \exp(x_{it}\beta)$. Moment conditions (1) are equivalent to

$$E\left(x_{it-s}\left(\frac{y_{it}}{\mu_{it}} - \frac{y_{it-1}}{\mu_{it-1}}\right)\right) = 0,$$

which are the moment conditions as originally proposed by Wooldridge (1991).[1] (Wooldridge, 1997, endnote 2) discarded these moment conditions, because if an x contains for example only nonnegative values, the associated β would go to infinity. A solution to this problem is to transform x_{it} in deviation of the overall mean

[1]Consider the moment conditions as proposed by Chamberlain,

$$E\left(x_{it-s}\left(y_{it}\frac{\mu_{it-1}}{\mu_{it}} - y_{it-1}\right)\right) = E(x_{it-s}\mu_{it-1}\Delta u_{it}).$$

Clearly, these moment conditions are valid only under weak exogeneity, not when x_{it} is endogenously determined.

$$\bar{x} = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} x_{it}.^{2}$$

The moment conditions (2) have not been considered before for the multiplicative setting, but are equivalent to the nonlinear moment conditions for the linear dynamic model as proposed by Ahn and Schmidt (1995). Given the stated assumptions, moment conditions (1) and (2) constitute the complete set of moment conditions.

3. Monte Carlo results

This section presents some illustrative Monte Carlo results. The data generating process is given by

$$y_{it} \sim \text{Poisson}(\exp(x_{it}\beta + \eta_i + \varepsilon_{it}))$$

$$x_{it} = \rho x_{it-1} + \delta\eta_i + \theta\varepsilon_{it} + \omega_{it}$$

$$\eta_i \sim N(0, \sigma^2_\eta);\ \varepsilon_{it} \sim N(0, \sigma^2_\varepsilon);\ \omega_{it} \sim N(0, \sigma^2_\omega),$$

and therefore x_{it} is endogenous when $\theta \neq 0$. Table 1 presents estimation results for four two-step

Table 1
Monte Carlo results for $T = 5$[a]

	$N = 1000$		$N = 2000$		$N = 5000$	
	Mean	S.D.	Mean	S.D.	Mean	S.D.
bch^{t-1}	0.783	0.120	0.785	0.077	0.789	0.052
bw^{t-1}	0.744	0.082	0.757	0.062	0.778	0.045
bch^{t-2}	0.611	0.270	0.641	0.167	0.664	0.070
bw^{t-2}	0.518	0.110	0.507	0.085	0.505	0.057
Sargan	Mean	$P < 0.05$	Mean	$P < 0.05$	Mean	$P < 0.05$
ch^{t-1}	23.17	0.793	37.40	0.989	79.00	1.000
w^{t-1}	23.73	0.757	37.18	0.980	72.28	1.000
ch^{t-2}	4.96	0.045	5.03	0.043	5.12	0.045
w^{t-2}	5.01	0.051	4.98	0.048	4.90	0.048

[a] $\beta = 0.5$; $\rho = 0.8$; $\delta = 0.1$; $\theta = 0.3$; $\sigma^2_\eta = 0.3$; $\sigma^2_\varepsilon = 0.25$; $\sigma^2_\omega = 0.3$.
Means and standard deviations for 1000 replications.
bch^j and bw^j are two-step GMM estimators using the Chamberlain or Wooldridge moments respectively, with instruments dated j and further lags. Number of moment conditions are 10 and 6 for $(t-1)$ and $(t-2)$ instruments, respectively.

[2] I found in several Monte Carlo experiments that this transformation improved efficiency when a regressor contained both negative and positive values but had non-zero mean.
Additionally, a stable way of estimating time effects is to specify the moment conditions as

$$E\left(z_{it}\left(\frac{y_{it}}{\mu_{it}} - \frac{\delta_t}{\delta_{t-1}}\frac{y_{it-1}}{\mu_{it-1}}\right)\right) = 0,$$

where z_{it} are the instruments, including time dummies.

GMM estimators, for $T = 5$ and $N = 1000$, 2000 and 5000. The first two use the Chamberlain (1992) or Wooldridge (1991) moment conditions respectively, with instruments dated $(t - 1)$ and further lags. The true value of β is 0.5, and both estimators are clearly substantially biased upwards. The moment conditions are rejected by the Sargan test for overidentifying restrictions for both estimators. The third and fourth estimators use instruments dated $(t - 2)$ and further lags.[3] It is clear that use of moment conditions (1) results in a consistent estimator, whereas trying to allow for endogeneity for Chamberlain's moment conditions by lagging the instruments still results in an inconsistent estimator. For *both* estimators, the Sargan statistic does not reject the moment conditions.

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[3] For this model specification, the extra moment conditions (2) only marginally improved efficiency, results are available from the author on request.